Heisenberg-Limited Uncertainty for Ultrafast Electromagnetic Pulses

The common form of the Heisenberg Uncertainty Principle, $\Delta x \cdot \Delta p_x \geq \frac{h}{4 \cdot \pi}$, is derived from the commutator relation: $\sigma_f \cdot \sigma_g \geq \frac{1}{2} \cdot |\langle [f,g] \rangle|$. Although time is not an operator, an energy-time relation can be drawn from the above formula:

$$\Delta E \cdot \Delta t \geq \frac{h}{4 \cdot \pi}$$

and from this, several other relationships,

$$\Delta \nu \cdot \Delta t \geq \frac{1}{4 \cdot \pi}$$ since frequency is proportional to energy, all energy relations can be derived from this

recall that $\nu = \frac{c}{\lambda}$, and let $\lambda_1 = \lambda_c + \frac{1}{2} \cdot w$ and $\lambda_2 = \lambda_c - \frac{1}{2} \cdot w$, where w is the full width, then

$$\Delta \nu = \frac{c \cdot w}{\lambda_c^2 - \frac{w^2}{4}} \quad \text{and} \quad \Delta t = \frac{\lambda_c^2 - \frac{w^2}{4}}{4 \cdot \pi \cdot c \cdot w}$$

Energy Solution

$$\Delta \nu \geq \frac{1}{4 \cdot \pi \cdot \Delta t} \qquad \Delta \nu(\Delta t) := \frac{1}{4 \cdot \pi \cdot \Delta t}$$

Wavelength Solution

$$w = \begin{pmatrix} -8 \cdot \pi \cdot c \cdot \Delta t + 2 \cdot \sqrt{16 \cdot \pi^2 \cdot c^2 \cdot \Delta t^2 + \lambda_c^2} \\ -8 \cdot \pi \cdot c \cdot \Delta t - 2 \cdot \sqrt{16 \cdot \pi^2 \cdot c^2 \cdot \Delta t^2 + \lambda_c^2} \end{pmatrix}$$ formula for width

$$w(\Delta t, \lambda_c) := -8 \cdot \pi \cdot c \cdot \Delta t + 2 \cdot \sqrt{16 \cdot \pi^2 \cdot c^2 \cdot \Delta t^2 + \lambda_c^2}$$

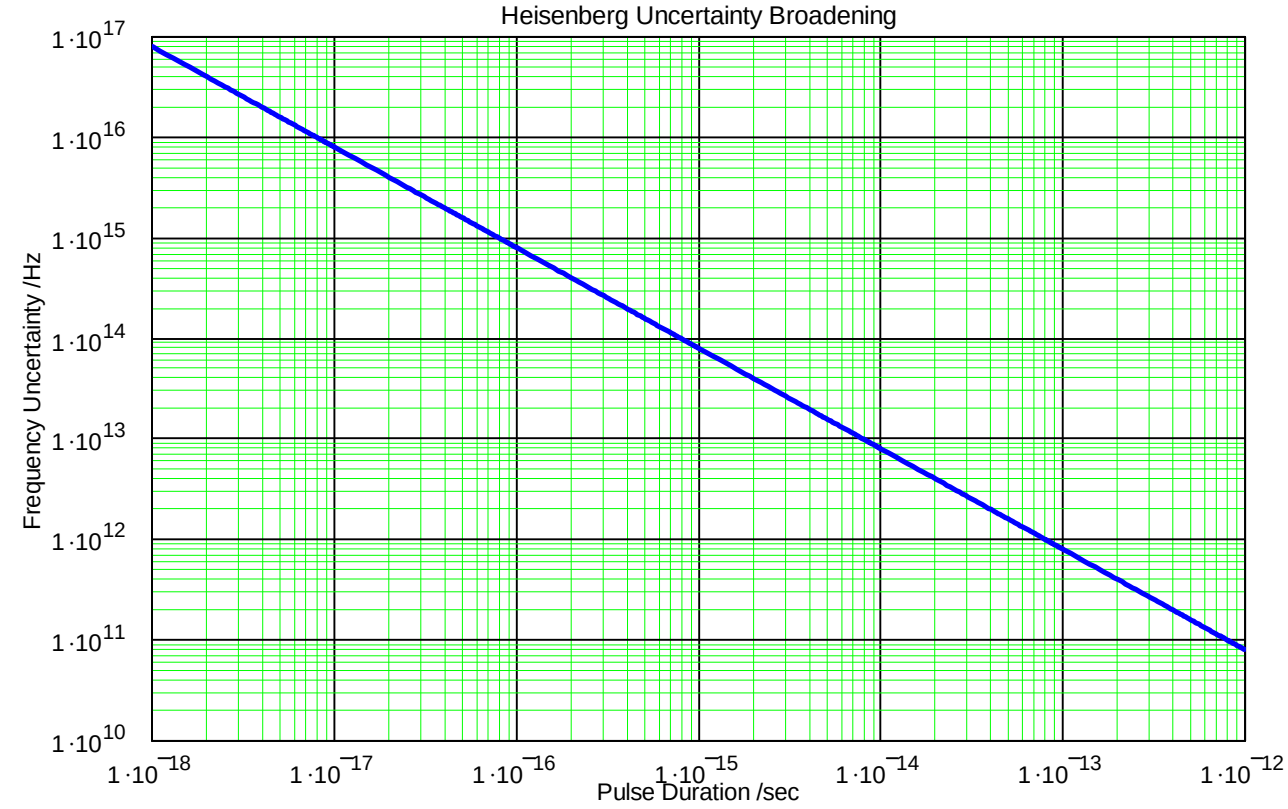
```
t := | offset ← 13
     | for i ∈ 0 .. 5
     |   for j ∈ 10 .. 100
     |     T_(100·i+j−10·(i+1)) ← (j/10)·10^−(i+offset)
     | sort(T)
```

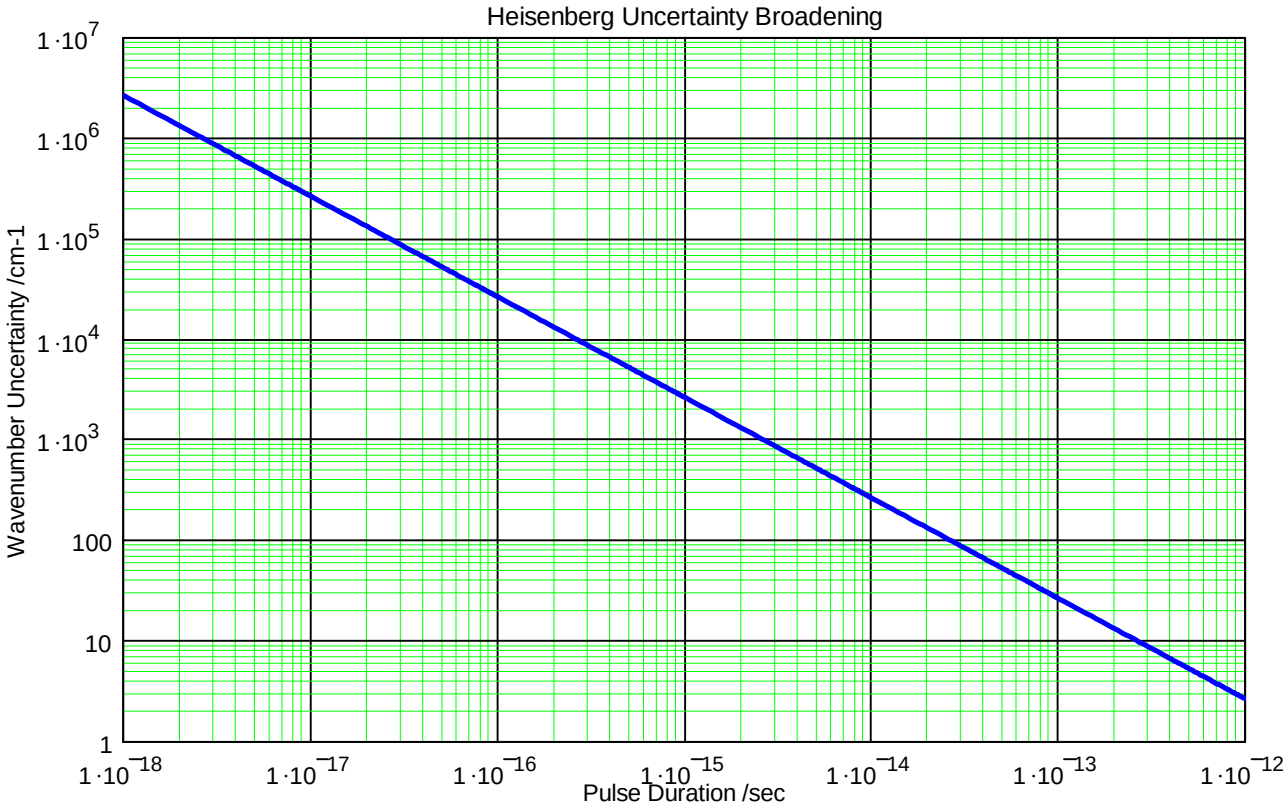
calculates 90 data points for each order of magnitude of time

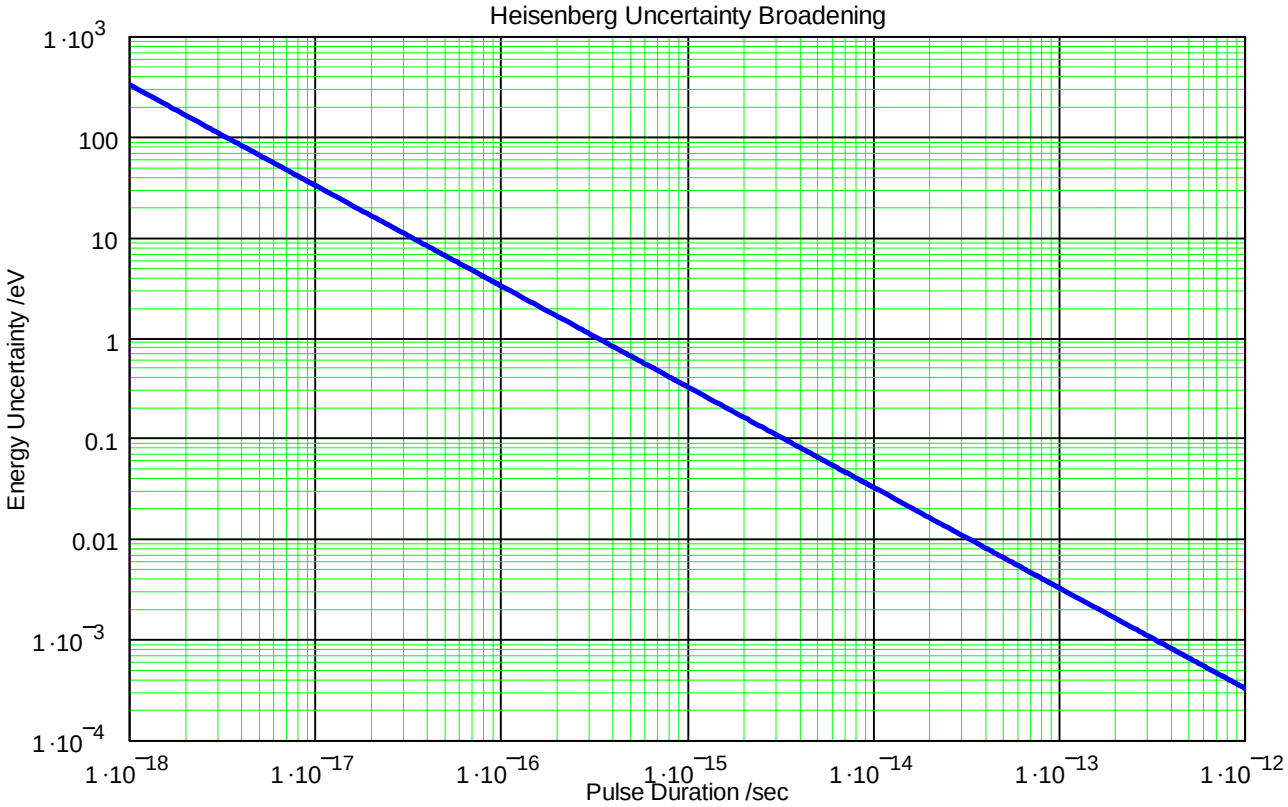
t := submatrix(t, 0, rows(t) − 1, 0, 0)

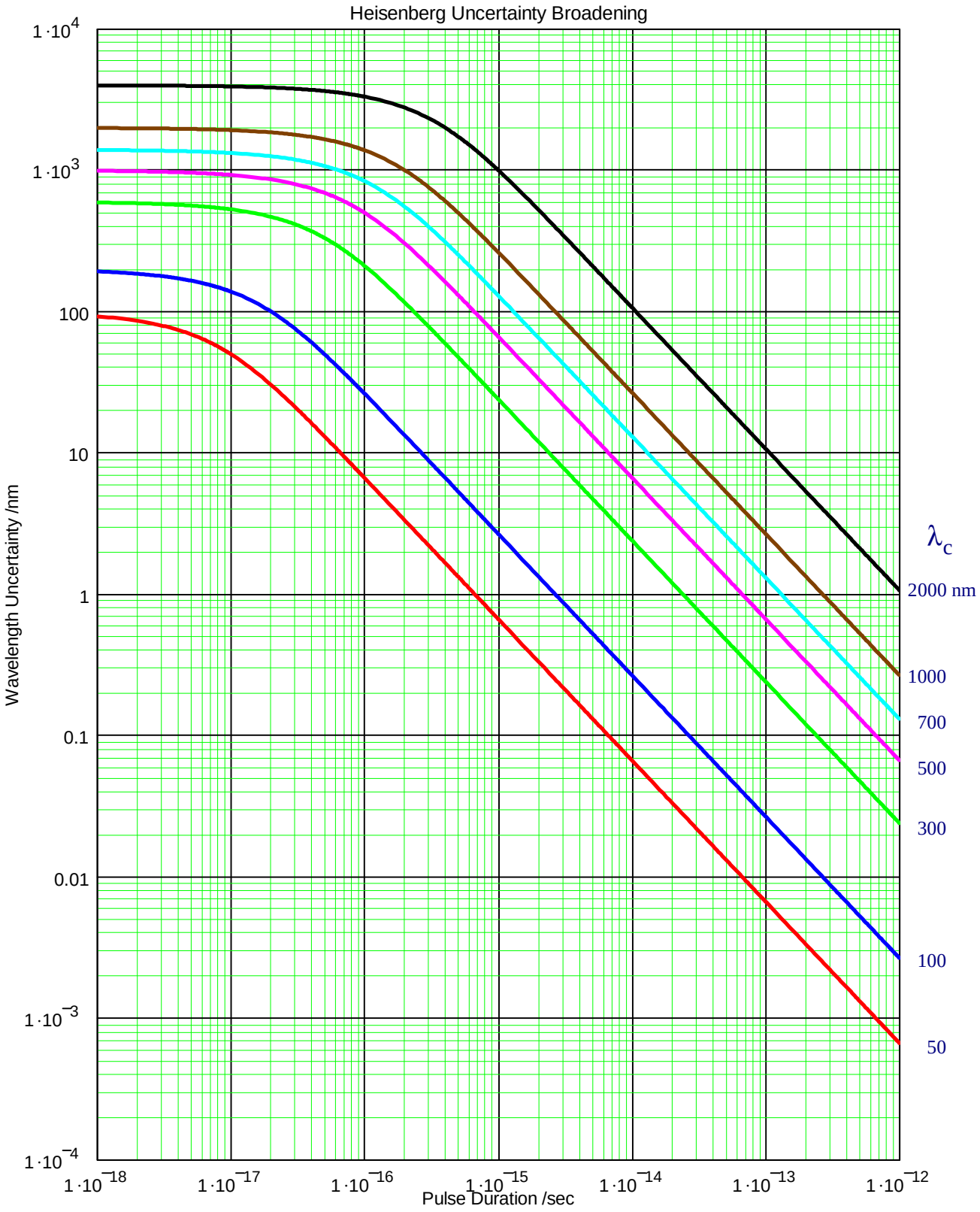
t := t·sec

i := 0 .. rows(t) − 1


Heisenberg Uncertainty Broadening
Frequency Uncertainty /Hz
1·10^17
1·10^16
1·10^15
1·10^14
1·10^13
1·10^12
1·10^11
1·10^10
1·10^-18
1·10^-17
1·10^-16
1·10^-15
1·10^-14
1·10^-13
1·10^-12
Pulse Duration /sec


Heisenberg Uncertainty Broadening
Wavenumber Uncertainty /cm-1
1·10^7
1·10^6
1·10^5
1·10^4
1·10^3
100
10
1
1·10^-18
1·10^-17
1·10^-16
1·10^-15
1·10^-14
1·10^-13
1·10^-12
Pulse Duration /sec


Heisenberg Uncertainty Broadening
Energy Uncertainty /eV
1·10^3
100
10
1
0.1
0.01
1·10^-3
1·10^-4
1·10^-18
1·10^-17
1·10^-16
1·10^-15
1·10^-14
1·10^-13
1·10^-12
Pulse Duration /sec


Heisenberg Uncertainty Broadening
Wavelength Uncertainty /nm
Pulse Duration /sec
λ_c
2000 nm
1000
700
500
300
100
50
$1 \cdot 10^{4}$
$1 \cdot 10^{3}$
100
10
1
0.1
0.01
$1 \cdot 10^{-3}$
$1 \cdot 10^{-4}$
$1 \cdot 10^{-18}$
$1 \cdot 10^{-17}$
$1 \cdot 10^{-16}$
$1 \cdot 10^{-15}$
$1 \cdot 10^{-14}$
$1 \cdot 10^{-13}$
$1 \cdot 10^{-12}$