# Heisenberg-Limited Uncertainty for Ultrafast Electromagnetic Pulses

The common form of the Heisenberg Uncertainty Principle, $\Delta x \cdot \Delta p_x \geq \frac{h}{4 \cdot \pi}$, is derived from the commutator relation: $\sigma_f \cdot \sigma_g \geq \frac{1}{2} \cdot |\langle [f,g] \rangle|$. Although time is not an operator, an energy-time relation can be drawn from the above formula:

$$\Delta E \cdot \Delta t \geq \frac{h}{4 \cdot \pi}$$

and from this, several other relationships,

$$\Delta \nu \cdot \Delta t \geq \frac{1}{4 \cdot \pi}$$

since frequency is proportional to energy, all energy relations can be derived from this

recall that $\nu = \frac{c}{\lambda}$, and let $\lambda_1 = \lambda_c + \frac{1}{2} \cdot w$ and $\lambda_2 = \lambda_c - \frac{1}{2} \cdot w$, where w is the full width, then

$$\Delta \nu = \frac{c \cdot w}{\lambda_c^{\ 2} - \frac{w^2}{4}} \quad \text{and} \quad \Delta t = \frac{\lambda_c^{\ 2} - \frac{w^2}{4}}{4 \cdot \pi \cdot c \cdot w}$$

## Energy Solution

$$\Delta \nu \geq \frac{1}{4 \cdot \pi \cdot \Delta t} \qquad \Delta \nu(\Delta t) := \frac{1}{4 \cdot \pi \cdot \Delta t}$$

## Wavelength Solution

$$w = \begin{pmatrix} -8 \cdot \pi \cdot c \cdot \Delta t + 2 \cdot \sqrt{16 \cdot \pi^2 \cdot c^2 \cdot \Delta t^2 + \lambda_c^{\ 2}} \\ -8 \cdot \pi \cdot c \cdot \Delta t - 2 \cdot \sqrt{16 \cdot \pi^2 \cdot c^2 \cdot \Delta t^2 + \lambda_c^{\ 2}} \end{pmatrix}$$

formula for width

$$w(\Delta t, \lambda_c) := -8 \cdot \pi \cdot c \cdot \Delta t + 2 \cdot \sqrt{16 \cdot \pi^2 \cdot c^2 \cdot \Delta t^2 + \lambda_c^{\ 2}}$$

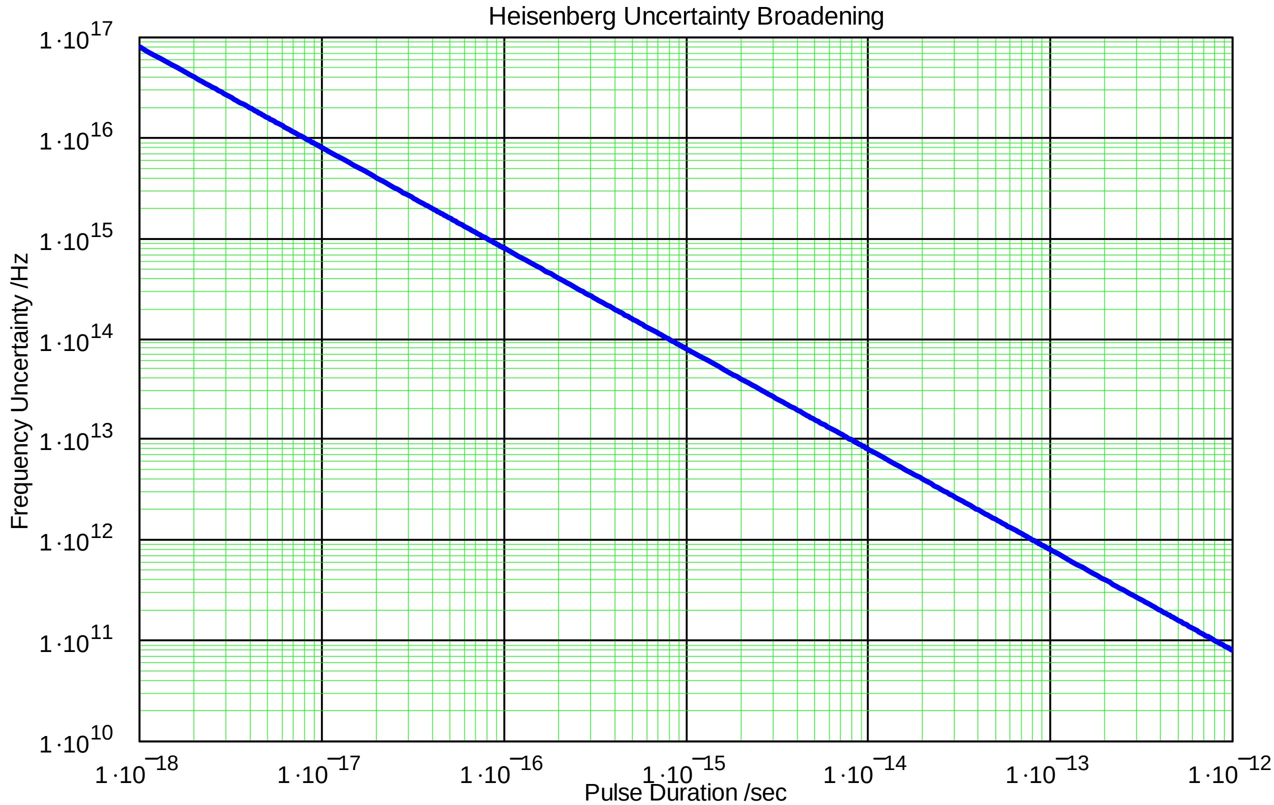
$$t := \left| \begin{array}{l} \text{offset} \leftarrow 13 \\ \text{for } i \in 0..5 \\ \quad \text{for } j \in 10..100 \\ \quad\quad T_{100\cdot i+j-10\cdot(i+1)} \leftarrow \frac{j}{10}\cdot 10^{-(i+\text{offset})} \\ \text{sort}(T) \end{array} \right.$$

calculates 90 data points for each order of magnitude of time

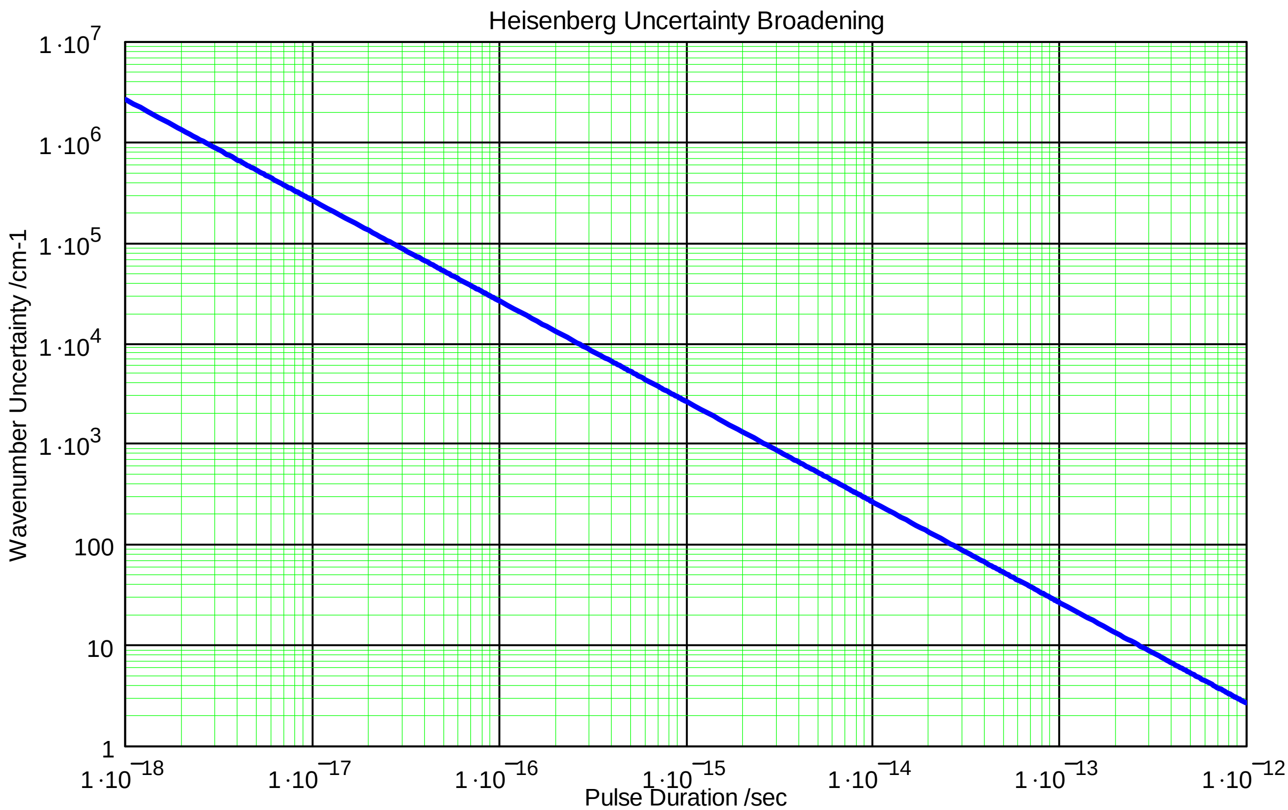
$t := \text{submatrix}(t, 0, \text{rows}(t) - 1, 0, 0)$

$t := t\cdot \text{sec}$

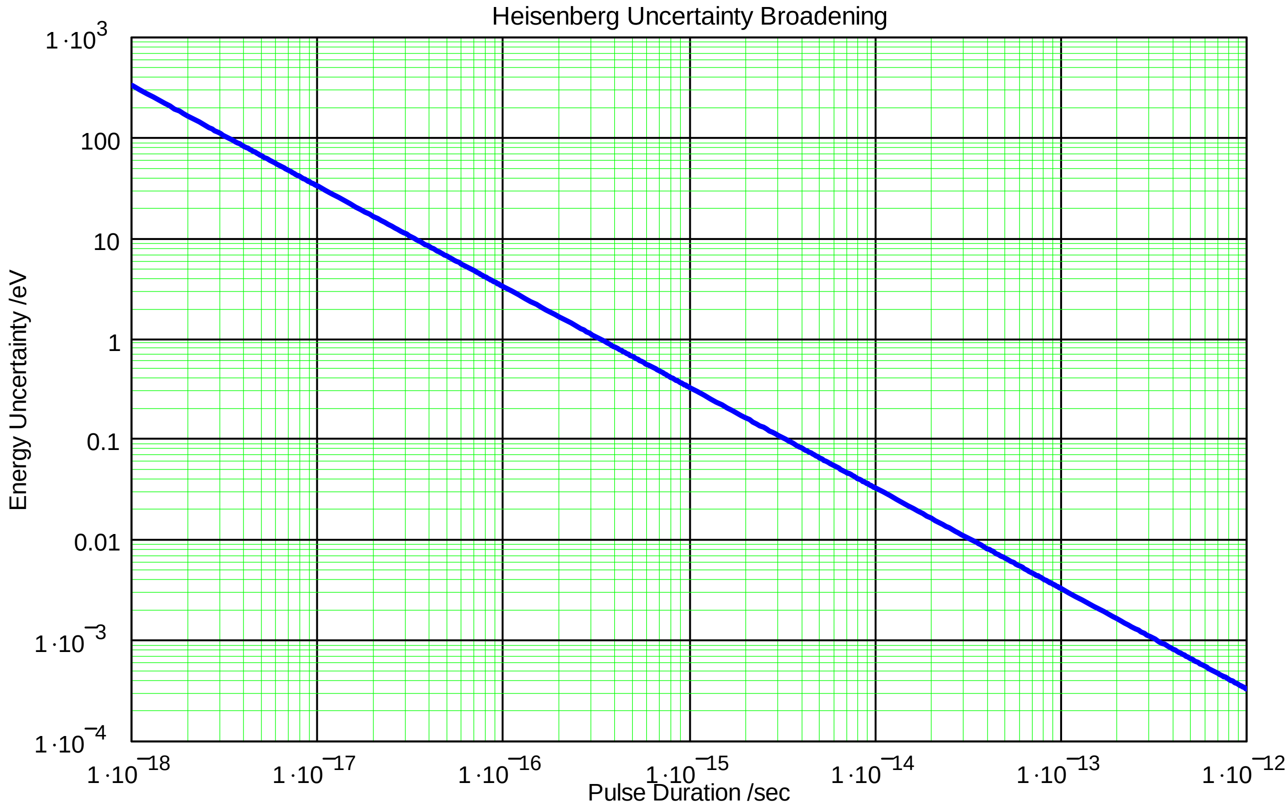
$i := 0 .. \text{rows}(t) - 1$

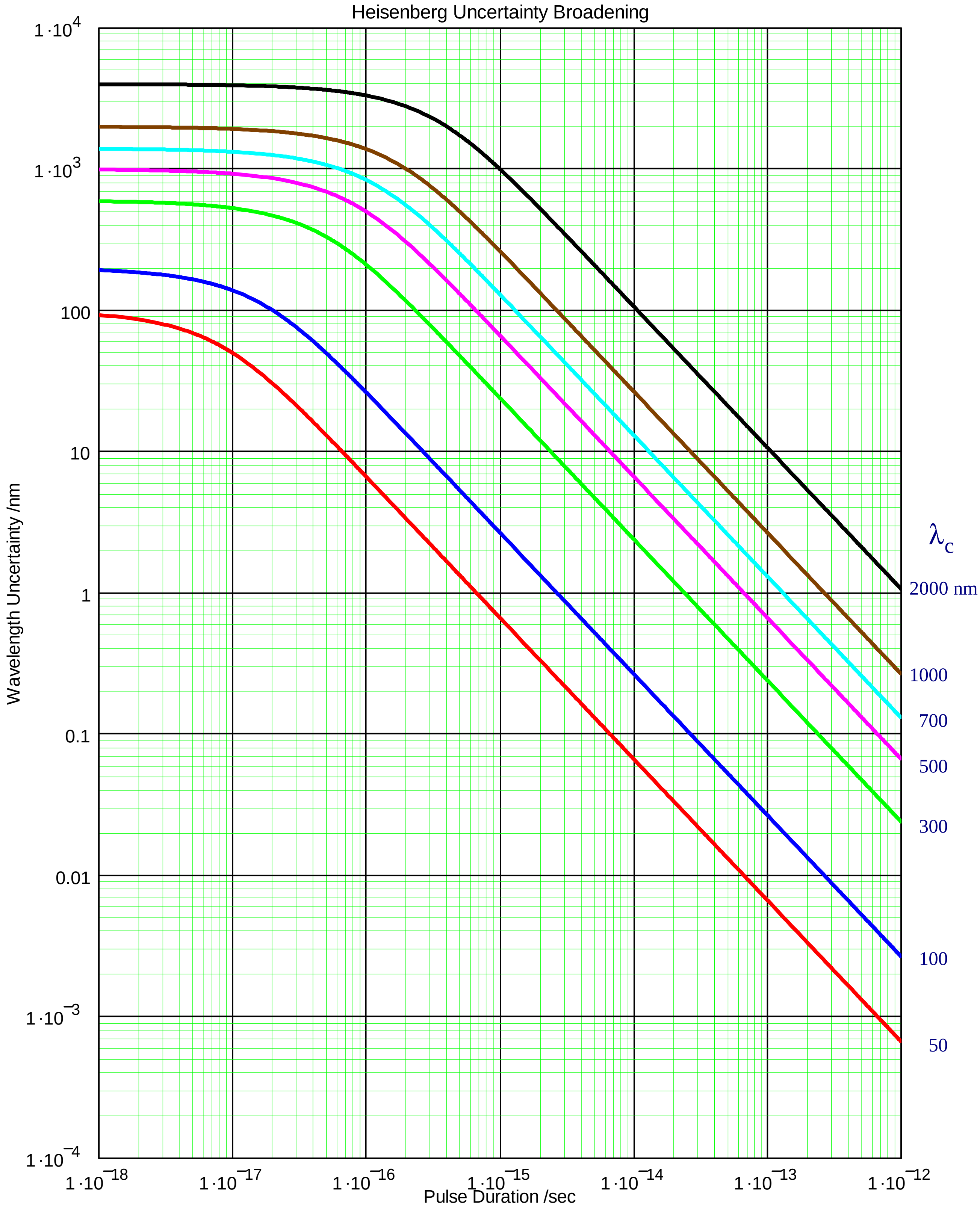


Heisenberg Uncertainty Broadening
Frequency Uncertainty /Hz
1·10^17
1·10^16
1·10^15
1·10^14
1·10^13
1·10^12
1·10^11
1·10^10
1·10^−18
1·10^−17
1·10^−16
1·10^−15
1·10^−14
1·10^−13
1·10^−12
Pulse Duration /sec

Heisenberg Uncertainty Broadening
Wavenumber Uncertainty /cm-1
1·10^7
1·10^6
1·10^5
1·10^4
1·10^3
100
10
1
1·10^−18
1·10^−17
1·10^−16
1·10^−15
1·10^−14
1·10^−13
1·10^−12
Pulse Duration /sec


Heisenberg Uncertainty Broadening
Energy Uncertainty /eV
Pulse Duration /sec
1·10^3
100
10
1
0.1
0.01
1·10^−3
1·10^−4
1·10^−18
1·10^−17
1·10^−16
1·10^−15
1·10^−14
1·10^−13
1·10^−12


Heisenberg Uncertainty Broadening
Wavelength Uncertainty /nm
Pulse Duration /sec
1·10^4
1·10^3
100
10
1
0.1
0.01
1·10^−3
1·10^−4
1·10^−18
1·10^−17
1·10^−16
1·10^−15
1·10^−14
1·10^−13
1·10^−12
λc
2000 nm
1000
700
500
300
100
50